Comment on Precursors in a Pressure Driven Shock Tube

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IN commenting on a technical note by W. Zinman,[1] Prof. Weymann[2] referred to our work[3] and incorrectly estimated our diaphragm-probe distance as 100 cm. As a consequence of this estimate, he concluded the following: "... that the two probes will measure a stationary state at x = 50 cm ahead of the shock front if the distance of the first probe from the diaphragm is larger than z_c = 96 cm ... It is, therefore, quite curious that Lederman and Wilson find stationary profiles in their shock tube."

The over-all length of the driven section is 10 ft, or 305 cm, as stated in Ref. 3. The correct diaphragm-probe distance is 210 cm. Therefore, even according to Professor Weymann's own analysis, it is not at all "curious" that we found stationary profiles in our experiments.

References

[1] Zinman, W., "Comment on Experimental Precursor Studies," *AIAA Journal*, Vol. 4, No. 11, 1966, pp. 2073–2075.

[2] Weymann, H., "Comments on Precursors ahead of Pressure Driven Shock Waves," *AIAA Journal*, Vol. 5, No. 7, 1967, pp. 1375–1376.

[3] Lederman, S. and Wilson, D. S., "Microwave Resonant Cavity Measurement of Shock Produced Electron Precursors," *AIAA Journal*, Vol. 5, No. 1, 1967, pp. 70–78.

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Comments on "Circulatory Flow of a Conducting Liquid about a Porous, Rotating Cylinder in a Radial Magnetic Field"

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A COUPLE of years ago,[1] we considered a class of exact, stationary, vortex-like solutions of incompressible, dissipative, MHD flow under the influence of a radial magnetic field, $-C/r$, and an azimuthal magnetic field. It was shown that the basic MHD equations can be reduced to a single ordinary differential equation. In terms of the Nomenclature of Ref. 2, this equation has the form:

$$[\xi^3(d^3/d\xi^3) + (4 + R + RP_m)\xi^2(d^2/d\xi^2) + (1 + 3R + RP_m + R^2P_m - Q)\xi(d/d\xi) - (1 - R + RP_m - R^2P_m + Q)]v_\theta = -2Qe_z\xi \quad (1)$$

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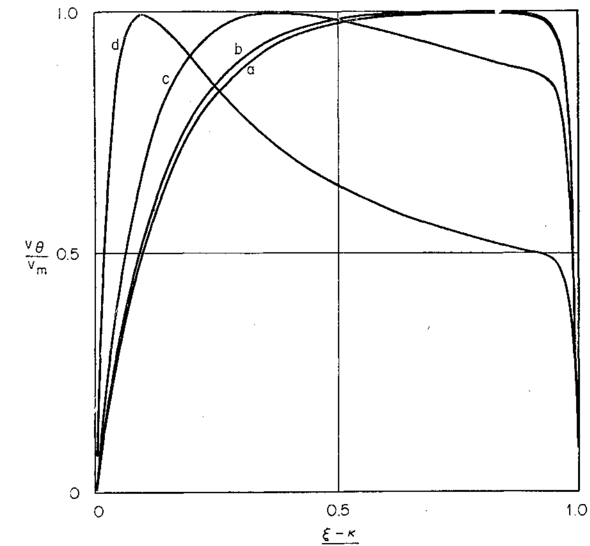

Fig. 1 MHD-driven circulatory flow between stationary cylinders (e_z = 0.5, R = −100, C = 1.0, κ = 0.25): a) Q = 1.0, v_m = 0.0593; b) Q = 10, v_m = 0.556; c) Q = 100, v_m = 3.14; d) Q = 1000, v_m = 3.93.

where $e_z = E_za^2/(\mu_eCD)$ and a is a characteristic radius. Most of our work pertains to the flow between concentric cylinders, and we choose the outer cylinder radius as a.

The solution of Eq. (1) is

$$v_\theta = A'\xi^{n_1} + B'\xi^{n_2} + C'\xi^{n_3} + G\xi \quad (2)$$

where $n_{1,2}$ are given by Eq. (16) of Ref. 2 and $n_3 = -1$. The constant G is given by

$$G = Qe_z/[Q - R(2 + RP_m)] \quad (3)$$

The constants A', B', C' are to be determined by the velocity and magnetic boundary conditions. The assumed radial components of the velocity and magnetic fields are the same as those of Eq. (8), Ref. 2.

Using Ohm's law and the equation of motion in the θ direction, h_θ can be obtained. The result is

$$h_\theta = \frac{A'RP_m}{n_1 + 1 + RP_m}\xi^{n_1} + \frac{B'RP_m}{n_2 + 1 + RP_m}\xi^{n_2} + C'\xi^{-1} + \frac{R^2P_me_z}{Q - R(2 + RP_m)}\xi \quad (4)$$

For the special case of flow around a rotating porous cylinder, the last term in Eq. (4) must be discarded. This leads to the same expression as that given by Eq. (15), Ref. 2.

In the limit as $RP_m \to 0$, the term $C'\xi^{-1}$ of Eqs. (2) and (4) remains finite if an azimuthal magnetic field is applied by passing an electric current through a conductor along the axis. The value of C' then becomes the ratio of the applied azimuthal magnetic field to the applied radial magnetic field. If RP_m is set equal to zero, Eq. (2) reduces to the expression given by Lewellen[3] [Eq. (28)] with the additional term $C'\xi^{-1}$. Thus, the present solution with $RP_m = 0$ is more general than that given in Ref. 3. If RP_m is set equal to zero in Eq. (4), only the applied field $C'\xi^{-1}$ remains. If the perturbation of h_θ due to the flow is needed, the terms in Eq. (4) of first order in RP_m must also be retained.

We have made a complete parametric study of the expression in Eq. (2) for small RP_m for flow between concentric cylinders driven by an impressed electric field E_z, as well as by the applied azimuthal magnetic field. Some typical curves of v_θ vs ξ with the extra term $C'\xi^{-1}$ included are given

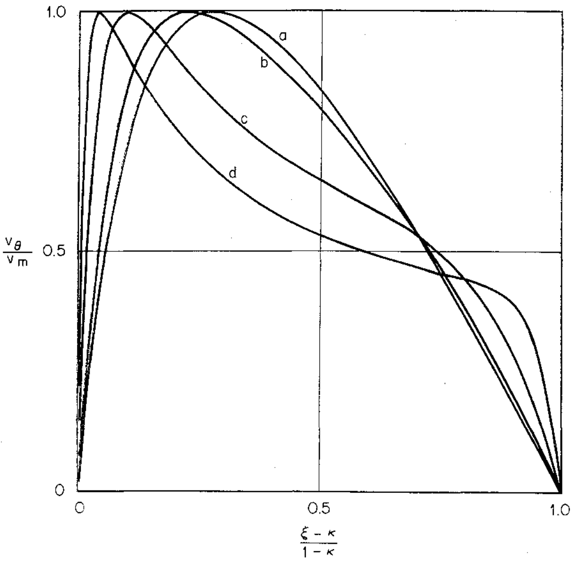

Fig. 2 MHD-driven circulatory flow between stationary cylinders (e_z = 0.5, R = 0, C = 1.0, κ = 0.25): a) Q = 1.0, v_m = 0.405; b) Q = 10, v_m = 1.77; c) Q = 100, v_m = 2.94; d) Q = 1000, v_m = 3.60.

in Figs. 1 through 3. In the figures, the velocity has been normalized by dividing by its maximum value v_m:

$$v_m = \max_{\kappa \leq \xi \leq 1} v_\theta \tag{5}$$

where κ is the ratio of inner to outer cylinder radius.

When Q is large (e.g., Q = 1000 in the figures), the velocity profiles approach the curve

$$v_\theta = (C'/\xi) + G\xi \tag{6}$$

in the interior of the fluid, where C' and G are determined by the applied fields and are independent of the velocitus of the confining cylinders. Boundary layers are formed near the cylinders. The effect of the imposed radial flow is to increase the thickness of the boundary layer near the injection cylinder and to decrease the thickness of the boundary layer near the suction cylinder.

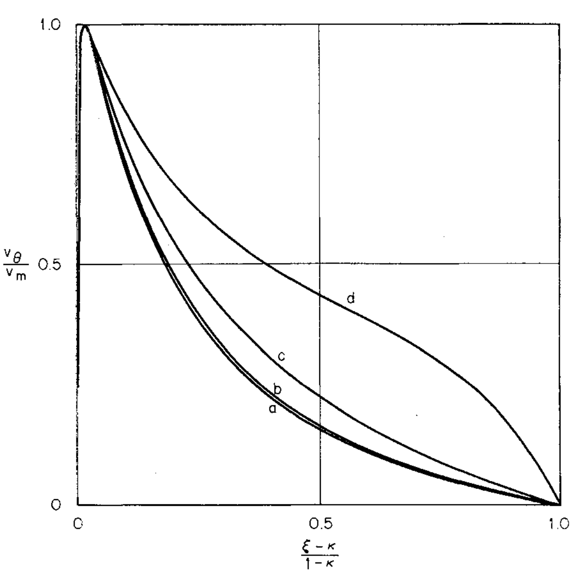

Fig. 3 MHD-driven circulatory flow between stationary cylinders (e_z = 0.5, R = 100, C = 1.0, κ = 0.25): a) Q = 1.0, v_m = 0.016; b) Q = 10, v_m = 0.152; c) Q = 100, v_m = 1.06; d) Q = 1000, v_m = 2.92.

There are a few minor misprints in Ref. 2: 1) Equation at the bottom of page 380 should be

$$-C\frac{d}{dr}(ru_\theta) - \frac{\mu_e D}{4\pi\rho}\frac{d}{dr}(rH_\theta) = \nu r^2\frac{d}{dr}\left(\frac{du_\theta}{dr} + \frac{u_\theta}{r}\right)$$

2) μ_2 in Eq. (11) should be μ_e; 3) $\eta_{1,2}$ in Eq. (16) should be $n_{1,2}$; 4) η_1 in Eq. (23) should be n_1; 5) B in Eq. (29) should be B_1; and 6) A in Eq. (31) should be A_1.

References

[1] Chang, T. S., "Radially Symmetric Motion of a Conducting Fluid Under the Influence of Radial and Transverse Magnetic Fields," AEC-ORNL-PR-1-12, Aug. 1965, unpublished Consultant Report, Oak Ridge National Laboratory, Oak Ridge, Tenn.

[2] Gupta, A. S., "Circulatory Flow of a Conducting Liquid About a Porous, Rotating Cylinder in a Radial Magnetic Field," *AIAA Journal*, Vol. 5, No. 2, Feb. 1967, pp. 380–382.

[3] Lewellen, W. S., "Magnetohydrodynamically Driven Vortices," *Proceedings of the Heat Transfer and Fluid Mechanics Institute*, 1960, pp. 1–15.

Addendum: "A Theoretical Investigation of MHD Channel Entrance Flows"

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EQUATION (5) of our paper was an asymptotic expression for the fully developed friction factor for turbulent MHD flow between parallel plates. The exponent of the denominator of the right-hand side of this equation should

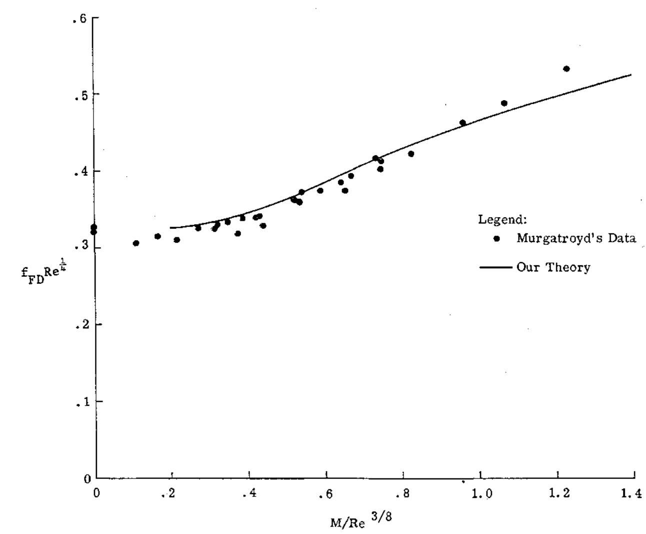

Fig. 1 Comparison of new theoretical expression [Eq. (2)] with experiment.

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